

Sample programs in C^{++} for matrix computations in max plus algebra

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Abstract. The main purpose of this paper is to propose five programs in C^{++} for matrix computations and solving recurrent equations systems with entries in max plus algebra. ¹

1 Introduction

Idempotent mathematics is based on replacing the usual arithmetic operations with a new set of basic operations, that is on replacing numerical fields by idempotent semirings. Exotic semirings such as the max-plus algebra \mathbf{R}_{max} have been introduced in connection with various fields: graph theory, Markov decision processes, discrete event systems theory, see [1], [3].

The paper is organized as follows. The semiring of matrices with entries in max plus algebra is presented in Section 2. In Section 3 we give five programs in language C^{++} for matrix computations in max plus algebra.

2 Semirings. Matrices over max plus algebra

We start this section by recalling of some necessary backgrounds on semirings for our purposes (see [3],[2] and references therein for more details).

Let S be a nonempty set endowed with two binary operations, *addition* (denoted with \oplus) and *multiplication* (denoted with \otimes). The algebraic structure $(S, \oplus, \otimes, \varepsilon, e)$ is a *semiring*, if it fulfills the following conditions:

- (1) (S, \oplus, ε) is a commutative monoid with ε as the neutral element for \oplus ;
- (2) (S, \otimes, e) is a monoid with ε as the identity element for \otimes ;
- (3) \otimes distributes over \oplus ;
- (4) ε is an absorbing element for \otimes , that is $a \otimes \varepsilon = \varepsilon \otimes a = \varepsilon$, $\forall a \in S$.

A semiring where addition is idempotent (that is, $a \oplus a = a$, $\forall a \in S$) is called an *idempotent semiring*. If \otimes is commutative, we say that S is a *commutative semiring*.

Let $(S, \oplus, \otimes, \varepsilon, e)$ be an (idempotent) semiring. For each pair of positive integer (m, n) , let $M_{m \times n}(S)$ be denote the set of $m \times n$ matrices with entries in S . The operations \oplus and \otimes on S induce corresponding operations on $M_{m \times n}(S)$ in the obvious way. Indeed, if $A = (A_{ij}), B = (B_{ij}) \in M_{m \times n}(S)$ then we have:

$$A \oplus B = ((A \oplus B)_{ij}) \quad \text{where} \quad (A \oplus B)_{ij} := A_{ij} \oplus B_{ij}. \quad (2.1)$$

If $A = (A_{ij}) \in M_{m \times n}(S)$ and $B = (B_{jk}) \in M_{n \times p}(S)$ then we have:

¹AMS classification: 15A80, 68-04.

Key words and phrases: idempotent semiring, max plus algebra.

$$A \otimes B = ((A \otimes B)_{ik}), \quad i = \overline{1, m}, \quad k = \overline{1, p} \quad \text{where} \quad (A \otimes B)_{ik} := \bigoplus_{j=1}^n A_{ij} \otimes B_{jk}. \quad (2.2)$$

The product of a matrix $A = (A_{ij}) \in M_{m \times n}(S)$ with a scalar $\alpha \in S$ is given by:

$$\alpha \otimes A = ((\alpha \otimes A)_{ij}) \quad \text{where} \quad (\alpha \otimes A)_{ij} := \alpha \otimes A_{ij}. \quad (2.3)$$

The set $M_{n \times n}(S)$ contains two special matrices with entries in S , namely the zero matrix $O_{\oplus n}$, which has all its entries equal to ε , and the identity matrix $I_{\otimes n}$, which has the diagonal entries equal to e and the other entries equal to ε .

It is easy to check that the following proposition holds.

Proposition 2.1. *$(M_{n \times n}(S), \oplus, \otimes, O_{\oplus n}, I_{\otimes n})$ is an idempotent semiring, where the operations \oplus and \otimes are given in (2.1) and (2.2).* \square

We call $(M_{n \times n}(S), \oplus, \otimes, O_{\oplus n}, I_{\otimes n})$ the *semiring of $n \times n$ matrices with entries in S* . In particular, if $S := \mathbf{R}_{max} = (\mathbf{R} \cup \{-\infty\}, \oplus := \max, \otimes := +, \varepsilon := -\infty, e := 0)$ is called the *semiring of $n \times n$ matrices over \mathbf{R}_{max}* .

When $S = \mathbf{R}_{max}$, the operations \oplus and \otimes given in (2.1) and (2.2), becomes:

$$(A \oplus B)_{ij} := \max\{A_{ij}, B_{ij}\} \quad \text{and} \quad (A \otimes B)_{ik} := \max_{1 \leq j \leq n} \{A_{ij} + B_{jk}\}. \quad (2.4)$$

The operation \otimes on $M_{m \times n}(\mathbf{R}_{max})$ given in (2.3) becomes:

$$\alpha \otimes A = ((\alpha \otimes A)_{ij}) \quad \text{where} \quad (\alpha \otimes A)_{ij} := \alpha + A_{ij}. \quad (2.5)$$

3 Five programs in C^{++}

In this section we give programs written in the language C^{++} for the basic operations with matrices over \mathbf{R}_{max} and for solving a recurrent linear system:

1. the sum of two matrices $A, B \in M_{m \times n}(\mathbf{R}_{max})$;
2. the product of two matrices $A \in M_{m \times n}(\mathbf{R}_{max})$ and $B \in M_{n \times p}(\mathbf{R}_{max})$;
3. the product of a matrix $A \in M_{m \times n}(\mathbf{R}_{max})$ with a scalar $\alpha \in \mathbf{R}_{max}$;
4. the power of a matrix $A \in M_{n \times n}(\mathbf{R}_{max})$;
5. the solving a linear system of the form:

$$X(k+1) = A \otimes X(k), \quad k \geq 0,$$

where $A \in M_{n \times n}(\mathbf{R}_{max})$ and $X(k) \in M(n, 1; \mathbf{R}_{max})$.

The principal program is constituted from the following lines.

```
using System;
using System.Collections.Generic;
```

```

using System.ComponentModel;
using System.Data;
using System.Drawing;
using System.Text;
using System.Windows.Forms;
namespace Operations_ with_matrices
{
    public partial class Form1 : Form
    {
        public Form1()
        {
            InitializeComponent();
        }
        public int[,] A5 = new int[50, 50];
        #region sum of two matrices in max plus algebra
        private void initMatrixA()
        {
            int column = 0;
            column = Convert.ToInt16(textColumn.Text);
            dataGridA.ColumnCount = column;
            dataGridA.AllowUserToOrderColumns = false;
            dataGridA.AllowUserToAddRows = false;
            dataGridA.Enabled = true;
            dataGridA.AutoSizeRowsMode = DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
            dataGridA.ColumnHeaderBorderStyle = DataGridViewHeaderBorderStyle.Raised;
            dataGridA.CellBorderStyle = DataGridViewCellBorderStyle.Single;
            dataGridA.GridColor = Color.DodgerBlue;
            dataGridA.ColumnHeadersVisible = false;
            dataGridA.RowHeadersVisible = false;
            dataGridA.BackgroundColor = Color.WhiteSmoke;
            dataGridA.BorderStyle = BorderStyle.None;
            dataGridA.AllowUserToResizeColumns = true;
            DataGridViewCellStyle style = new DataGridViewCellStyle();
            style.Format = "N0";
            style.NullValue = null;
            DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
            columnHeaderStyle.BackColor = Color.AntiqueWhite;
            columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
            dataGridA.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
            DataGridViewCell CellR = new DataGridViewTextBoxCell();

```

```

CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
DataGridViewCell CellL = new DataGridViewTextBoxCell();
CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
int line = 0;
line = Convert.ToInt16(textLine.Text);
dataGridA.RowCount = line;
for (int i = 0; i < column; i++)
{
    dataGridA.Columns[i].Name = "C" + (i + 1);
    dataGridA.Columns[i].CellTemplate = CellR;
    dataGridA.Columns[i].Width = 30;
    dataGridA.Columns[i].DefaultCellStyle = style;
}
}
private void initMatrixB()
{
    int column = 0;
    column = Convert.ToInt16(textColumn.Text);
    dataGridB.ColumnCount = column;
    dataGridB.AllowUserToOrderColumns = false;
    dataGridB.AllowUserToAddRows = false;
    dataGridB.Enabled = true;
    dataGridB.AutoSizeRowsMode = DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridB.ColumnHeaderBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridB.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridB.GridColor = Color.DodgerBlue;
    dataGridB.ColumnHeadersVisible = false;
    dataGridB.RowHeadersVisible = false;
    dataGridB.BackgroundColor = Color.WhiteSmoke;
    dataGridB.BorderStyle = BorderStyle.None;
    dataGridB.AllowUserToResizeColumns = true;

    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridB.ColumnHeadersDefaultCellStyle = columnHeaderStyle;

    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;

```

```

    int line = 0;
    line = Convert.ToInt16(textLine.Text);
    dataGridB.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridB.Columns[i].Name = "C" + (i + 1);
        dataGridB.Columns[i].CellTemplate = CellR;
        dataGridB.Columns[i].Width = 30;
    }
}

private void initMatrix of Addition Results ()
{
    int column = 0;
    column = Convert.ToInt16(textColumn.Text);
    dataGridResAddition.ColumnCount = column;
    dataGridResAddition.AllowUserToOrderColumns = false;
    dataGridResAddition.AllowUserToAddRows = false;
    dataGridResAddition.Enabled = true;
    dataGridResAddition.AutoSizeRowsMode = DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridResAddition.ColumnHeadersBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridResAddition.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridResAddition.GridColor = Color.DodgerBlue;
    dataGridResAddition.ColumnHeadersVisible = false;
    dataGridResAddition.RowHeadersVisible = false;
    dataGridResAddition.BackgroundColor = Color.WhiteSmoke;
    dataGridResAddition.BorderStyle = BorderStyle.None;
    dataGridResAddition.AllowUserToResizeColumns = true;

    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridResAddition.ColumnHeadersDefaultCellStyle = columnHeaderStyle;

    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;

    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;

    int line = 0;
    line = Convert.ToInt16(textLine.Text);
    dataGridResAddition.RowCount = line;
    for (int i = 0; i < column; i++)

```

```

    {
        dataGridResAddition.Columns[i].Name = "C" + (i + 1);
        dataGridResAddition.Columns[i].CellTemplate = CellR;
        dataGridResAddition.Columns[i].Width = 30;
    }
}
private void btGenerate_Click(object sender, EventArgs e)
{
    if (textLine.Text != "" && textColumn.Text != "")
    {
        initMatrixA();
        initMatrixB();
    }
    else
    {
        if (textLine.Text == "" && textColumn.Text == "")
            MessageBox.Show("Introduce number of lines and number of columns!");
        else if (textLine.Text == "")
            MessageBox.Show("Introduce number of lines!");
        else if (textColumns.Text == "")
            MessageBox.Show("Introduce number of columns!");
    }
}
private void btComputeSumClick(object sender, EventArgs e)
{
    initMatrixAdditionResult();
    int line = 0;
    int column = 0;
    column = Convert.ToInt16(textColumn.Text);
    line = Convert.ToInt16(textLine.Text);
    int[,] A = new int[line, column];
    int[,] B = new int[line, column];
    int[,] ResAddition = new int[line, column];
    # region reading matrix A
    for (int i = 0; i < line; i++)
    {
        for (int j = 0; j < column; j++)
        {
            if (dataGridA.Rows[i].Cells[j].Value.ToString() == "E")
                A[i, j] = Int32.MinValue;
            else

```

```

        A[i, j] = Convert.ToInt16(dataGridA.Rows[i].Cells[j].Value.ToString());
        if (dataGridB.Rows[i].Cells[j].Value.ToString() == "E")
            B[i, j] = Int32.MinValue;
        else
            B[i, j] = Convert.ToInt16(dataGridB.Rows[i].Cells[j].Value.ToString());
    }
}
# endregion
# region computation of sum
for (int i = 0; i < line; i++)
{
    for (int j = 0; j < column; j++)
    {
        if (A[i, j] > B[i, j])
            ResAddition[i, j] = A[i, j];
        else
            ResAddition[i, j] = B[i, j];
    }
}
# endregion
# region display sum
for (int i = 0; i < line; i++)
{
    for (int j = 0; j < column; j++)
    {
        if (ResAddition[i, j] == Int32.MinValue)
            dataGridResAddition.Rows[i].Cells[j].Value = "E";
        else
            dataGridResAddition.Rows[i].Cells[j].Value = ResAddition[i, j];
    }
}
# endregion
}
public void Reset_Values_For_Addition()
{
    textColumn.ResetText();
    textLine.ResetText();
    dataGridA.Rows.Clear();
    dataGridB.Rows.Clear();
    dataGridResAddition.Rows.Clear();
}
private void btReset_Click(object sender, EventArgs e)

```

```

{
    Reset_Values _For_Addition();
}
# endregion

# region product of two matrices in max plus algebra
private void initMatrixA2()
{
    int column = 0;
    column = Convert.ToInt16(textColumnA2.Text);
    dataGridA2.ColumnCount = column;
    dataGridA2.AllowUserToOrderColumns = false;
    dataGridA2.AllowUserToAddRows = false;
    dataGridA2.Enabled = true;
    dataGridA2.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridA2.ColumnHeadersBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridA2.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridA2.GridColor = Color.DodgerBlue;
    dataGridA2.ColumnHeadersVisible = false;
    dataGridA2.RowHeadersVisible = false;
    dataGridA2.BackgroundColor = Color.WhiteSmoke;
    dataGridA2.BorderStyle = BorderStyle.None;
    dataGridA2.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridA2.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
    int line = 0;
    line = Convert.ToInt16(textLineA2.Text);
    dataGridA2.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridA2.Columns[i].CellTemplate = CellR;
        dataGridA2.Columns[i].Width = 30;
    }
}

```



```

private void initMatrixB2()
{
    int column = 0;
    column = Convert.ToInt16(textcolumnB2.Text);
    dataGridB2.ColumnCount = column;
    dataGridB2.AllowUserToOrderColumns = false;
    dataGridB2.AllowUserToAddRows = false;
    dataGridB2.Enabled = true;
    dataGridB2.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridB2.ColumnHeadersBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridB2.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridB2.GridColor = Color.DodgerBlue;
    dataGridB2.ColumnHeadersVisible = false;
    dataGridB2.RowHeadersVisible = false;
    dataGridB2.BackgroundColor = Color.WhiteSmoke;
    dataGridB2.BorderStyle = BorderStyle.None;
    dataGridB2.AllowUserToResizeColumns = true;

    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridB2.ColumnHeadersDefaultCellStyle = columnHeaderStyle;

    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;

    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;

    int line = 0;
    line = Convert.ToInt16(textLineB2.Text);
    dataGridB2.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridB2.Columns[i].Name = "C" + (i + 1);
        dataGridB2.Columns[i].CellTemplate = CellR;
        dataGridB2.Columns[i].Width = 30;
    }
}
private void initMatrixProduct()
{
    int column = 0;
    column = Convert.ToInt16(textcolumnB2.Text);

```

```

dataGridProduct.ColumnCount = column;
dataGridProduct.AllowUserToOrderColumns = false;
dataGridProduct.AllowUserToAddRows = false;
dataGridProduct.Enabled = true;
dataGridProduct.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
dataGridProduct.ColumnHeadersBorderStyle = DataGridViewHeaderBorderStyle.Raised;
dataGridProduct.CellBorderStyle = DataGridViewCellBorderStyle.Single;
dataGridProduct.GridColor = Color.DodgerBlue;
dataGridProduct.ColumnHeadersVisible = false;
dataGridProduct.RowHeadersVisible = false;
dataGridProduct.BackgroundColor = Color.WhiteSmoke;
dataGridProduct.BorderStyle = BorderStyle.None;
dataGridProduct.AllowUserToResizeColumns = true;
DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
columnHeaderStyle.BackColor = Color.AntiqueWhite;
columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
dataGridProduct.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
DataGridViewCell CellR = new DataGridViewTextBoxCell();
CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
DataGridViewCell CellL = new DataGridViewTextBoxCell();
CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
int line = 0;
line = Convert.ToInt16(textLineA2.Text);
dataGridProduct.RowCount = line;
for (int i = 0; i < column; i++)
{
    dataGridProduct.Columns[i].Name = "C" + (i + 1);
    dataGridProduct.Columns[i].CellTemplate = CellR;
    dataGridProduct.Columns[i].Width = 30;
}
}
private void textColumnA2_Leave(object sender, EventArgs e)
{
    textLineB2.Text = textColumnA2.Text;
}
private void btGenerare2_Click(object sender, EventArgs e)
{
    if (textLineA2.Text != "" && textColumnA2.Text != "" && textLineB2.Text !=
"" && textcolumnB2.Text != "")

```

```

    {
        initMatrixA2();
        initMatrixB2();
    }
    else
    {
        if (textLineA2.Text == "" && textColumnA2.Text == "" && textLineB2.Text
== "" && textcolumnnoB2.Text == "")
            MessageBox.Show("Introduce number of lines and number of columns!");
        else
        {
            if (textLineA2.Text == "" || textLineB2.Text == "")
                MessageBox.Show("Introduce number of lines for matrix A!");
            else if (textColumnA2.Text == "")
                MessageBox.Show("Introduce number of columns for matrix A!");
            else if (textcolumnnoB2.Text == "")
                MessageBox.Show("Introduce number of columns for matrix B!");
        }
    }
}
private void btComputationProduct_Click(object sender, EventArgs e)
{
    initMatrixProduct();
    int lineA = 0; int lineB = 0;
    int columnA = 0; int columnB = 0;
    columnA = Convert.ToInt16(textColumnA2.Text);
    lineA = Convert.ToInt16(textLineA2.Text);
    columnB = Convert.ToInt16(textcolumnB2.Text);
    lineB = Convert.ToInt16(textLineB2.Text);
    int[,] A2 = new int[lineA, columnA];
    int[,] B2 = new int[lineB, columnB];
    int[,] Product = new int[lineA, columnB];
    int[,] Sum = new int[lineA, columnA];
    int k;
    # region reading matrices A and B
    for (int i = 0; i < lineA; i++)
    {
        for (int j = 0; j < columnA; j++)
        {
            if (dataGridA2.Rows[i].Cells[j].Value.ToString() == "E")
                A2[i, j] = Int32.MinValue;

```

```

    else
        A2[i, j] = Convert.ToInt16(dataGridA2.Rows[i].Cells[j].Value.ToString());
    }
}
for (int i = 0; i < lineB; i++)
{
    for (int j = 0; j < columnB; j++)
    {
        if (dataGridB2.Rows[i].Cells[j].Value.ToString() == "E")
            B2[i, j] = Int32.MinValue;
        else
            B2[i, j] = Convert.ToInt16(dataGridB2.Rows[i].Cells[j].Value.ToString());
        }
    }
}
# endregion
# region computation product
for (int i = 0; i < lineA; i++) {
    for (int j = 0; j < columnB; j++)
    {
        Product[i, j] = Int32.MinValue;
        for (k = 0; k < lineB; k++)
        {
            if (A2[i, k] == Int32.MinValue || B2[k, j] == Int32.MinValue)
                Sum[i, j] = Int32.MinValue;
            else
                Sum[i, j] = A2[i, k] + B2[k, j];
            if (Product[i, j] > Sum[i, j])
                Product[i, j] = Sum[i, j];
            else
                Product[i, j] = Sum[i, j];
        }
    }
}
# endregion
# region display product
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < columnB; j++)
    {
        if (Product[i, j] == Int32.MinValue)
            dataGridProduct.Rows[i].Cells[j].Value = "E";
    }
}

```

```

        else
            dataGridProduct.Rows[i].Cells[j].Value = Product[i, j];
        }
    }
    # endregion
}
public void Reset_Values for Multiplication()
{
    textColumnA2.ResetText();
    textLineA2.ResetText();
    textcolumnB2.ResetText();
    textLineB2.ResetText();
    dataGridA2.Rows.Clear();
    dataGridB2.Rows.Clear();
    dataGridProduct.Rows.Clear();
}
private void btResetMultiplication_Click(object sender, EventArgs e)
{
    Reset_Values_for_Multiplication ();
}
# endregion

#region scalar product in max plus algebra
private void initMatrixA3()
{
    int column = 0;
    column = Convert.ToInt16(textcolumnA3.Text);
    dataGridA3.ColumnCount = column;
    dataGridA3.AllowUserToOrderColumns = false;
    dataGridA3.AllowUserToAddRows = false;
    dataGridA3.Enabled = true;
    dataGridA3.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridA3.ColumnHeaderBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridA3.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridA3.GridColor = Color.DodgerBlue;
    dataGridA3.ColumnHeadersVisible = false;
    dataGridA3.RowHeadersVisible = false;
    dataGridA3.BackgroundColor = Color.WhiteSmoke;
    dataGridA3.BorderStyle = BorderStyle.None;
    dataGridA3.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();

```

```

columnHeaderStyle.BackColor = Color.AntiqueWhite;
columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
dataGridA3.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
DataGridViewCell CellR = new DataGridViewTextBoxCell();
CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
DataGridViewCell CellL = new DataGridViewTextBoxCell();
CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
int line = 0;
line = Convert.ToInt16(textLineA3.Text);
dataGridA3.RowCount = line;
for (int i = 0; i < column; i++)
{
    dataGridA3.Columns[i].Name = "C" + (i + 1);
    dataGridA3.Columns[i].CellTemplate = CellR;
    dataGridA3.Columns[i].Width = 30;
}
}
private void initMatrixScalarProduct()
{
    int column = 0;
    column = Convert.ToInt16(textcolumnA3.Text);
    dataGridScalarProduct.ColumnCount = column;
    dataGridScalarProduct.AllowUserToOrderColumns = false;
    dataGridScalarProduct.AllowUserToAddRows = false;
    dataGridScalarProduct.Enabled = true;
    dataGridScalarProduct.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridScalarProduct.ColumnHeadersBorderStyle = DataGridViewHeaderBorder-
Style.Raised;
    dataGridScalarProduct.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridScalarProduct.GridColor = Color.DodgerBlue;
    dataGridScalarProduct.ColumnHeadersVisible = false;
    dataGridScalarProduct.RowHeadersVisible = false;
    dataGridScalarProduct.BackgroundColor = Color.WhiteSmoke;
    dataGridScalarProduct.BorderStyle = BorderStyle.None;
    dataGridScalarProduct.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridScalarProduct.ColumnHeadersDefaultCellStyle = columnHeaderStyle;

```

```

DataGridViewCell CellR = new DataGridViewTextBoxCell();
CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
DataGridViewCell CellL = new DataGridViewTextBoxCell();
CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
int line = 0;
line = Convert.ToInt16(textLineA3.Text);
dataGridScalarProduct.RowCount = line;
for (int i = 0; i < column; i++)
{
    dataGridScalarProduct.Columns[i].Name = "C" + (i + 1);
    dataGridScalarProduct.Columns[i].CellTemplate = CellR;
    dataGridScalarProduct.Columns[i].Width = 30;
}
}
private void Generating butons3_Click(object sender, EventArgs e)
{
    if (textLineA3.Text != "")
    {
        initMatrixA3();
    }
    else
        MessageBox.Show("Introduce number of lines and number of columns!");
}
private void btScalarProduct_Click(object sender, EventArgs e) {
    initMatrixScalarProduct ();
    if (textScalar.Text != "")
    {
        int lineA = 0;
        int columnA = 0;
        columnA = Convert.ToInt16(textcolumnA3.Text);
        lineA = Convert.ToInt16(textLineA3.Text);
        int[,] A3 = new int[lineA, columnA];
        int[,] ScalarProduct = new int[lineA, columnA];
        int a;
        #region reading of scalar a and matrix A
        a = Convert.ToInt16(textScalar.Text);
        for (int i = 0; i < lineA; i++)
        {
            for (int j = 0; j < columnA; j++)
            {
                if (dataGridA3.Rows[i].Cells[j].Value.ToString() == "E")

```

```

        A3[i, j] = Int32.MinValue;
    else
        A3[i, j] = Convert.ToInt16(dataGridA3.Rows[i].Cells[j].Value.ToString());
    }
}
#endregion
#region computation of scalar product
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < columnA; j++)
    {
        if (A3[i, j] == Int32.MinValue)
            Scalar Product [i, j] = Int32.MinValue;
        else
            Scalar Product [i, j] = (A3[i, j] + a);
    }
}
#endregion
#region display scalar product
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < columnA; j++)
    {
        if (ScalarProduct [i, j] == Int32.MinValue)
            dataGridScalarProduct.Rows[i].Cells[j].Value = "E";
        else
            dataGridScalarProduct.Rows[i].Cells[j].Value = ScalarProduct[i,j];
    }
}
#endregion
}
}
public void Reset_Values_for_Scalar Multiplication()
{
    textcolumnA3.ResetText();
    textScalar.ResetText();
    textLineA3.ResetText();
    dataGridA3.Rows.Clear();
    dataGridScalarProduct.Rows.Clear();
}
private void btReset_Scalar_Product_Click(object sender, EventArgs e)

```



```

{
    Reset_Values_for_Scalar_Multiplication();
}
#endregion

#region Power of a matrix in max plus algebra
private void initMatrixA4() {
    int column = 0;
    column = Convert.ToInt16(textlineA4.Text);
    dataGridA4.ColumnCount = column;
    dataGridA4.AllowUserToOrderColumns = false;
    dataGridA4.AllowUserToAddRows = false;
    dataGridA4.Enabled = true;
    dataGridA4.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridA4.ColumnHeaderBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridA4.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridA4.GridColor = Color.DodgerBlue;
    dataGridA4.ColumnHeadersVisible = false;
    dataGridA4.RowHeadersVisible = false;
    dataGridA4.BackgroundColor = Color.WhiteSmoke;
    dataGridA4.BorderStyle = BorderStyle.None;
    dataGridA4.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridA4.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
    int line = 0;
    line = Convert.ToInt16(textlineA4.Text);
    dataGridA4.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridA4.Columns[i].Name = "C" + (i + 1);
        dataGridA4.Columns[i].CellTemplate = CellR;
        dataGridA4.Columns[i].Width = 30;
    }
}

```

```

private void initPowerMatrix ()
{
    int column = 0;
    column = Convert.ToInt16(textlineA4.Text);
    dataGridMatrix_at_power_n.ColumnCount = column;
    dataGridMatrix_at_power_n.AllowUserToOrderColumns=false;
    dataGridMatrix_at_power_n.AllowUserToAddRows=false;
    dataGridMatrix_at_power_n.Enabled =true;
    dataGridMatrix_at_power_n.AutoSizeRowsMode=
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridMatrix_at_power_n.ColumnHeadersBorderStyle =DataGridViewHeader-
BorderStyle.Raised;
    dataGridMatrix_at_power_n.CellBorderStyle =DataGridViewCellBorderStyle.Single;
    dataGridMatrix_at_power_n.GridColor =Color.DodgerBlue;
    dataGridMatrix_at_power_n.ColumnHeadersVisible=false;
    dataGridMatrix_at_power_n.RowHeadersVisible=false;
    dataGridMatrix_at_power_n.BackgroundColor =Color.WhiteSmoke;
    dataGridMatrix_at_power_n.BorderStyle =BorderStyle.None;
    dataGridMatrix_at_power_n.AllowUserToResizeColumns=true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridMatrix_at_power_n.ColumnHeadersDefaultCellStyle =columnHeaderStyle;
    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
    int line = 0;
    line = Convert.ToInt16(textlinieA4.Text);
    dataGridMatrix_at_power_n.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridMatrix_at_power_n.Columns[i].Name = "C" + (i +1);
        dataGridMatrix_at_power_n.Columns[i].CellTemplate =CellR;
        dataGridMatrix_at_power_n.Columns[i].Width =30;
    }
}
private void btGenerareA4_Click(object sender, EventArgs e)
{
    if (textlineA4.Text != "")

```

```

    {
        initMatrixA4();
    }
    else
        MessageBox.Show("Introduce number of lines and columns!"); }
private void btComputPower_Click(object sender, EventArgs e)
{
    initMatrixPower ();
    labelPower.Text = textPower.Text;
    if (textPower.Text != "")
    {
        int lineA = 0;
        int columnA = 0;
        columnA = Convert.ToInt16(textlineA4.Text);
        lineA = Convert.ToInt16(textlineA4.Text);
        int[,] A4 = new int[lineA, columnA];
        int[,] B = new int[lineA, columnA];
        int[,] Power_n = new int[lineA, columnA];
        int[,] sum = new int[lineA, columnA];
        int a;
        a = Convert.ToInt16(textPower.Text);
        #region reading matrix A
        for (int i = 0; i < lineA; i++)
        {
            for (int j = 0; j < columnA; j++)
            {
                if (dataGridA4.Rows[i].Cells[j].Value.ToString() == "E")
                    A4[i, j] = Int32.MinValue;
                else
                    A4[i, j] = Convert.ToInt16(dataGridA4.Rows[i].Cells[j].Value.ToString());
                B[i, j] = A4[i, j];
            }
        }
        #endregion
        #region PowerMatrix A
        for (int p = 2; p <= a; p++)
        {
            for (int i = 0; i < lineA; i++)
            {
                for (int j = 0; j < columnA; j++)
                {
                    Power_n[i, j] = Int32.MinValue;

```

```

        for (int k = 0; k < lineA; k++)
        {
            if (A4[i, k] == Int32.MinValue ——— B[k, j] == Int32.MinValue)
                sum[i, j] = Int32.MinValue;
            else
                sum[i, j] = A4[i, k] + B[k, j];
            if (Power_n[i, j] > sum[i, j])
                Power_n[i, j] = sum[i, j];
            else
                Power_n[i, j] = sum[i, j];
        }
    }
}
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < columnA; j++)
    {
        B[i, j] = Power_n[i, j];
    }
}
}
#endregion
#region dysplay matrix
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < columnA; j++)
    {
        if (Power_n[i, j] == Int32.MinValue)
            dataGridMatrix_at_power_n.Rows[i].Cells[j].Value = "E";
        else
            dataGridMatrix_at_Power_n.Rows[i].Cells[j].Value = Power_n[i, j];
    }
}
#endregion
}
public void Reset_Values_for_Lifting_at_Power()
{
    textlineA4.ResetText();
    textPower.ResetText();
    dataGridA4.Rows.Clear();
}

```

```

    dataGridMatrix_at_Power_n.Rows.Clear();
}
private void btReset_lifting_at_power_Click(object sender, EventArgs e)
{
    Reset_Values_for_Lifting_at_Power();
}
#endregion

#region Solving equations system in max plus algebra
private void initMatrixA5()
{
    int column = 0;
    column = Convert.ToInt16(textlineA6.Text);
    dataGridA5.ColumnCount = column;
    dataGridA5.AllowUserToOrderColumns = false;
    dataGridA5.AllowUserToAddRows = false;
    dataGridA5.Enabled = true;
    dataGridA5.AutoSizeRowsMode =
DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridA5.ColumnHeaderBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridA5.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridA5.GridColor = Color.DodgerBlue;
    dataGridA5.ColumnHeadersVisible = false;
    dataGridA5.RowHeadersVisible = false;
    dataGridA5.BackgroundColor = Color.WhiteSmoke;
    dataGridA5.BorderStyle = BorderStyle.None;
    dataGridA5.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridA5.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MidRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MidLeft;
    int line = 0;
    line = Convert.ToInt16(textlineA5.Text);
    dataGridA5.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridA5.Columns[i].Name = "C" + (i + 1);
    }
}

```

```

        dataGridA5.Columns[i].CellTemplate = CellR;
        dataGridA5.Columns[i].Width = 30;
    }
}
private void initMatrixX0()
{
    int column = 0;
    column = 1;
    dataGridX0.ColumnCount = column;
    dataGridX0.AllowUserToOrderColumns = false;
    dataGridX0.AllowUserToAddRows = false;
    dataGridX0.Enabled = true;
    dataGridX0.AutoSizeRowsMode =
    DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridX0.ColumnHeadersBorderStyle = DataGridViewHeaderBorderStyle.Raised;
    dataGridX0.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridX0.GridColor = Color.DodgerBlue;
    dataGridX0.ColumnHeadersVisible = false;
    dataGridX0.RowHeadersVisible = false;
    dataGridX0.BackgroundColor = Color.WhiteSmoke;
    dataGridX0.BorderStyle = BorderStyle.None;
    dataGridX0.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridX0.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
    int line = 0;
    line = Convert.ToInt16(textlineA5.Text);
    dataGridX0.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridX0.Columns[i].Name = "C" + (i + 1);
        dataGridX0.Columns[i].CellTemplate = CellR;
        dataGridX0.Columns[i].Width = 30;
    }
}

```

```

private void initMatrixSolutionSystem()
{
    int column = 0;
    column = 1;
    dataGridSolutionXk.ColumnCount = column;
    dataGridSolutionXk.AllowUserToOrderColumns = false;
    dataGridSolutionXk.AllowUserToAddRows = false;
    dataGridSolutionXk.Enabled = true;
    dataGridSolutionXk.AutoSizeRowsMode =
        DataGridViewAutoSizeRowsMode.DisplayedCellsExceptHeaders;
    dataGridSolutionXk.ColumnHeadersBorderStyle = DataGridViewHeaderBorder-
Style.Raised;
    dataGridSolutionXk.CellBorderStyle = DataGridViewCellBorderStyle.Single;
    dataGridSolutionXk.GridColor = Color.DodgerBlue;
    dataGridSolutionXk.ColumnHeadersVisible = false;
    dataGridSolutionXk.RowHeadersVisible = false;
    dataGridSolutionXk.BackgroundColor = Color.WhiteSmoke;
    dataGridSolutionXk.BorderStyle = BorderStyle.None;
    dataGridSolutionXk.AllowUserToResizeColumns = true;
    DataGridViewCellStyle columnHeaderStyle = new DataGridViewCellStyle();
    columnHeaderStyle.BackColor = Color.AntiqueWhite;
    columnHeaderStyle.Alignment = DataGridViewContentAlignment.MiddleCenter;
    dataGridSolutionXk.ColumnHeadersDefaultCellStyle = columnHeaderStyle;
    DataGridViewCell CellR = new DataGridViewTextBoxCell();
    CellR.Style.Alignment = DataGridViewContentAlignment.MiddleRight;
    DataGridViewCell CellL = new DataGridViewTextBoxCell();
    CellL.Style.Alignment = DataGridViewContentAlignment.MiddleLeft;
    int line = 0;
    line = Convert.ToInt16(textlineA5.Text);
    dataGridSolutionXk.RowCount = line;
    for (int i = 0; i < column; i++)
    {
        dataGridSolutionXk.Columns[i].Name = "C" + (i + 1);
        dataGridSolutionXk.Columns[i].CellTemplate = CellR;
        dataGridSolutionXk.Columns[i].Width = 30;
    }
}
private void btGenerateMatrices_Click(object sender, EventArgs e)
{
    if (textlineA5.Text != "")

```

```

    {
        initMatrixA5();
        initMatrixX0();
    }
    else
        MessageBox.Show("Introduce number of lines and columns!");
}
private void btComputSystem_Click(object sender, EventArgs e)
{
    initMatrixSolutionSystem();
    label_k.Text = textk.Text;
    if (textk.Text != "")
    {
        int lineA = 0;
        int columnA = 0;
        int linex0 = 0;
        columnA = Convert.ToInt16(textlinieA5.Text);
        lineA = Convert.ToInt16(textlinieA5.Text);
        linex0 = Convert.ToInt16(textlinieA5.Text);
        int[,] A5 = new int[lineA, columnA];
        int[,] X0 = new int[linex0, 1];
        int[,] Xk = new int[lineA, 1];
        int[,] B = new int[lineA, columnA];
        int[,] Power_n = new int[lineA, columnA];
        int[,] sum = new int[lineA, columnA];
        int[,] sum2 = new int[lineA, 1];
        int k;
        k = Convert.ToInt16(textk.Text);
        #region readings matrix A and matrix X0
        for (int i = 0; i < lineA; i++)
        {
            for (int j = 0; j < coloanaA; j++)
            {
                if (dataGridViewA5.Rows[i].Cells[j].Value.ToString() == "E")
                    A5[i, j] = Int32.MinValue;
                else
                    A5[i, j] = Convert.ToInt16(dataGridViewA5.Rows[i].Cells[j].Value.ToString());
                B[i, j] = A5[i, j];
            }
        }
        for (int i = 0; i < linex0; i++)
        {

```



```

    if (dataGridX0.Rows[i].Cells[0].Value.ToString() == "E")
        X0[i, 0] = Int32.MinValue;
    else
        X0[i, 0] = Convert.ToInt16(dataGridX0.Rows[i].Cells[0].Value.ToString());
}
#endregion
#region lifting at power k of matrix A
if (k == 1)
{
    for (int i = 0; i < lineA; i++)
    {
        for (int j = 0; j < columnA; j++)
        {
            Power_n[i, j] = A5[i, j];
        }
    }
}
else
{
    for (int p = 2; p <= k; p++)
    {
        for (int i = 0; i < lineA; i++)
        {
            for (int j = 0; j < columnA; j++)
            {
                Power_n[i, j] = Int32.MinValue;
                for (int h = 0; h < lineA; h++)
                {
                    if (A5[i, h] == Int32.MinValue || B[h, j] == Int32.MinValue)
                        sum[i, j] = Int32.MinValue;
                    else
                        sum[i, j] = A5[i, h] + B[h, j];
                    if (Power_n[i, j] > sum[i, j])
                        Power_n[i, j] = Power_n[i, j];
                    else
                        Power_n[i, j] = sum[i, j];
                }
            }
        }
    }
}
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < columnA; j++)

```

```

        {
            B[i, j] = Power_n[i, j];
        }
    }
}
}
#endregion
#region computation of matrix X(k)
for (int i = 0; i < lineA; i++)
{
    for (int j = 0; j < 1; j++)
    {
        Xk[i, j] = Int32.MinValue;
        for (int h = 0; h < lineA; h++)
        {
            if (Power_n[i, h] == Int32.MinValue || X0[h, j] == Int32.MinValue)
                sum2[i, j] = Int32.MinValue;
            else
                sum2[i, j] = Power_n[i, h] + X0[h, j];
            if (Xk[i, j] > sum2[i, j])
                Xk[i, j] = Xk[i, j];
            else
                Xk[i, j] = sum2[i, j];
        }
    }
}
#endregion
#region display of matrix X(k)
for (int i = 0; i < lineA; i++)
{
    if (Xk[i, 0] == Int32.MinValue)
        dataGridSolutionXk.Rows[i].Cells[0].Value = "E";
    else
        dataGridSolutionXk.Rows[i].Cells[0].Value = Xk[i, 0];
}
#endregion
}
}
public void Reset_Values_for_Computation_system()
{
    textlineA5.ResetText();
}

```

```

    textk.ResetText();
    label_k.Text = "k";
    dataGridSolutionXk.Rows.Clear();
    dataGridA5.Rows.Clear();
    dataGridX0.Rows.Clear();
}
private void btResetSystem_Click(object sender, EventArgs e)
{
    Reset_Values_for_Computation_System();
}
#endregion
}
}

```

We illustrate the utilization of the above programs in the following cases.

1. Sum of two matrices A and B in \mathbf{R}_{max}

Inputs data

Number of lines 3

Number of columns 4

Matrix A

3	E	8	-2
6	0	4	-9
E	5	-7	1

Matrix B

9	9	-1	-5
2	-1	6	-3
1	2	4	-5

Outputs data

Matrix $A \oplus B$

9	9	8	-2
6	0	6	-3
1	5	4	1

2. Product of two matrices A and B in \mathbf{R}_{max}

Inputs data

Number of lines of A ! 3

Number of lines of B ! 4

Number of columns of A ! 4

Number of columns of matrix B ! 3

Matrix A

2	1	-1	4
E	0	5	-3
-4	-2	E	6

Matrix B

5	0	1
7	4	E
-5	9	2
8	-6	1

Outputs data

Matrix $A \otimes B$

12	8	5
7	14	7
14	2	7

3. Multiplication with scalar a of a matrix A in \mathbf{R}_{max}

Inputs data

Number of lines for matrix $A!$

Number of columns for matrix $A!$

Scalar a

Matrix A

4	-7	8	2	E
5	E	0	E	8
9	2	E	3	1

Outputs data

Matrix $a \otimes A$

0	-11	4	-2	E
1	E	-4	E	4
5	-2	E	-1	-3

4. Power of a matrix A in \mathbf{R}_{max}

Inputs data

Number of lines and columns for matrix $A!$

Power of matrix $A!$

Matrix A

1	0	-2	E	3
0	2	E	4	1
1	-1	-4	5	3
7	9	4	3	0
8	0	-2	0	E

Outputs data

Matrix $A^{(9)}$

50	52	47	46	47
53	55	50	56	53
54	56	51	57	54
59	61	56	55	52
52	52	47	52	49

5. Solving equations system in \mathbf{R}_{max}

Inputs data

Number of lines and columns for matrix $A!$

Value for $k!$

Matrix $A!$

3	-5	-9	2
4	8	7	4
-6	E	0	E
1	1	E	2

Matrix $X(0)$

4
3
2
1

Outputs data

Matrix $X(10)$

70
83
56
76

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